The Transmission of TE_{01} Wave in Helix Waveguides*

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Summary—Relations are investigated between the transmission characteristics of a helix waveguide and its surface impedance in regions where any simple approximate formulas are not available because of the magnitude of the surface impedance. The numerical calculations show that, for any given value of the surface impedance and the angular mode index, there exist an infinite number of different modes which are distinguishable from each other by different values of the radial propagation content.

Selecting a mode with minimum attenuation for each given surface impedance, we can draw the equiattenuation lines, connecting these points of equal attenuation on the complex surface impedance plane. At some point on the complex surface impedance plane, a maximum value of the minimum attenuation is found. For the TM_0 mode supported by a helix waveguide 50 mm in diameter, used at a frequency of 50 kmc, this minimax value of the attenuation constant is about 8 neper per meter, and the corresponding value of the surface impedance is about 57.6−j28.8 ohms. The attenuation constants of all the TM_0 modes corresponding to this optimum value of the surface impedance cannot be smaller than this minimax value.

The same kind of calculations are also performed for the two lowest hybrid modes. Physical structures giving the best value of the surface impedance are also suggested.

Introduction

For the TE_{01} wave transmitted inside the cylindrical metallic waveguide, attenuation is very small and therefore transmission over a long distance is possible. However, because economic requirements limit the size of the waveguide which can be put to practical use to the same extent as is the ordinary telephone cable size, it is necessary to use a frequency in the millimeter wave range in order to realize the small attenuation.

For a guide with a two inch diameter such that the TE_{01} wave can be transmitted at an attenuation of two db/mile at the frequency 50 kmc, approximately 200 unwanted modes can also be transmitted. Therefore any deviation of the waveguide from a straight circular cylinder gives rise not only to an increase in attenuation due to mode conversion but also to signal distortions by mode conversion and reconversion into the original TE_{01} wave.[1]

The principal problem of the TE_{01} wave transmission is the elimination of the transformation between the TE_{01} mode and unwanted modes. Several ways of combating mode conversion effects have been proposed.[2–8] Among these, use of the spaced-ring and helical structures as a waveguide or a mode filter is most attractive.

Analyses of these structures have been done by several authors. Morgan and Young[2] have studied the transmission characteristics of a special type of helix waveguide which is composed of a sheath helix with a lossy jacket. They performed extensive numerical calculations and gave sufficient basis for a design of this type of helix waveguide. Recently, Unger[9] studied helix waveguides with a multilayer jacket and stated the numerical results for some guide dimensions and material properties. There remains, however, a possibility that other types of helix waveguides may have better characteristics. A way of studying this possibility is to analyze the helix waveguide as one having an anisotropic surface impedance. This approach of analysis is very general because any special helix structure can be characterized by a surface impedance properly assumed. Formal equations expressing the relations between the t-transmission characteristics and the surface impedance have already been obtained by Karbowiak[10] Hosono[3] and Piefke.[4] Unfortunately these equations cannot be used directly, especially when the magnitude of the surface impedance is large.

The object of the present paper is to show by numerical calculation, the direct relationship between t-transmission characteristics and surface impedance in regions where the simple approximate formulas are not available.

Characteristic Equation for a Zero-Pitch Helix Waveguide

The helix waveguide of radius a and pitch angle Ψ is

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[1] S. E. Miller, "Waveguide as a communication medium," *Bell Sys. Tech. J.*, vol. 33, p. 1209; November, 1954.

[2] S. P. Morgan and J. A. Young, "Helix waveguide," *Bell Sys. Tech. J.*, vol. 35, pp. 1347–1384; November, 1956.

[3] T. Hosono, "Helix waveguide," Inst. Elect. Commun. Engs. of Japan, Session of Microwave Transmission; February, 1957.

[4] G. Piefke, "Wellenausbreitung in der Sheiben-Leitung," *Arch. elekt. Übertragung*, vol. 11, pp. 49–59; February, 1957.

[5] Enrique A. Marcatili, "Heat loss in grooved metallic surface," Proc. IRE, vol. 45, pp. 1134–1139; August, 1957.

[6] T. Hosono and S. Kohno, "The transmission of TE_{01} wave in corrugated waveguides," *Congres Internationale, Circuits et Antennes Hyperfrequences*, Paris, France, Session 9, 54. vol. 1, p. 253; October, 1957.

[7] J. A. Morrison, "Heat loss of circular electric waves in helix waveguides," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 173–177; April, 1958.

[8] D. Marcuse, "Attenuation of TE_{01} wave within the curved helix waveguide, *Bell Sys. Tech. J.*, vol. 37, p. 1599; November, 1958.

[9] H.-G. Unger, "Helix waveguide theory and applications," *Bell Sys. Tech. J.*, vol. 37, p. 1649; November, 1958.

[10] A. E. Karbowiak, "Theory of imperfect waveguides: the effect of wall impedance," *J. Inst. Elec. Engr.*, vol. 102, pp. 698–708; September, 1955.

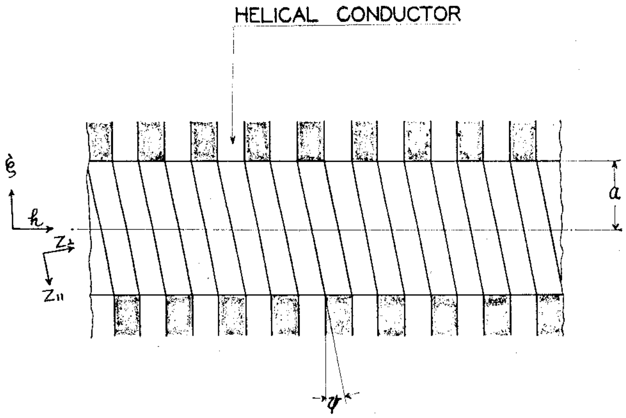

Fig. 1—The longitudinal section of a helix waveguide.

shown in Fig. 1. The characteristic equation for this type of helix waveguide is given by the following formula.[3,6]

$$\frac{\left(1-\frac{nh}{\xi^2 a}\cot\Psi+\frac{Z_\parallel}{Z_0}\frac{jkaJ'n(\xi a)}{\xi aJn(\xi a)}\right)\left(\frac{jkaJ'n(\xi a)}{\xi aJn(\xi a)}+\frac{Z_\perp}{Z_0}\left(1-\frac{nh}{\xi^2 a}\cot\Psi\right)\right)}{\left(1+\frac{nh}{\xi^2 a}\tan\Psi+\frac{Z_\perp}{Z_0}\frac{jkaJ'n(\xi a)}{\xi aJn(\xi a)}\right)\left(\frac{jkaJ'n(\xi a)}{\xi aJn(\xi a)}+\frac{Z_\parallel}{Z_0}\left(1+\frac{nh}{\xi^2 a}\tan\Psi\right)\right)}=\cot^2\Psi \tag{1}$$

where

propagation factor $\exp(jhz+jn\theta-j\omega t)$ is assumed and
a = inner radius of waveguide
h = axial propagation constant
$k=\omega\sqrt{\mu_0\epsilon_0}$ = propagation constant in free space
n = angular mode index
$Z_0=\sqrt{\mu_0/\epsilon_0}$ wave impedance of free space
$Z_\parallel$ = surface impedance parallel to the helix direction
$Z_\perp$ = surface impedance perpendicular to the helix
Ψ = pitch angle of helix
$\xi=\sqrt{k^2-h^2}$ = radial propagation constant.

When the pitch angle is very small (1) is reduced to

$$\left(\frac{kaJ'n(\xi a)}{\xi aJn(\xi a)}-j\frac{Z_\parallel}{Z_0}\right)\left(\frac{kaJ'n(\xi a)}{\xi aJn(\xi a)}-j\frac{Z_0}{Z_\perp}\right)=\left(\frac{nha}{\xi^2a^2}\right)^2. \tag{2}$$

This is the characteristic equation for a zero-pitch helix waveguide.[3,4,6] For $n=0$ (2) can be factored into two factors, and the solutions are

$$ka\frac{Z_0}{Z_\parallel}=-j\frac{(\xi a)J_0(\xi a)}{J_1(\xi a)} \tag{3}$$

for the TE_0 modes,

$$ka\frac{Z_\perp}{Z_0}=-j\frac{(\xi a)J_0(\xi a)}{J_1(\xi a)} \tag{4}$$

for the TM_0 modes.

These two modes are the only pure TE and TM modes that can exist in a zero-pitch helix waveguide. The TE_0 modes present low loss, and the TM_0 modes high loss. All other modes ($n\neq 0$) are mixed modes that cannot be separated into the pure TE or TM modes, so they are called hybrid modes, *i.e.*, HY_n modes. All the hybrid modes might also have high loss. For these modes, assuming $Z_\parallel/Z_0\ll 1$, we get the following formulas from (5):

$$\frac{ka}{\xi a}\frac{J'n(\xi a)}{Jn(\xi a)}\left(\frac{ka}{\xi a}\frac{J'n(\xi a)}{Jn(\xi a)}-j\frac{Z_0}{Z_\perp}\right)=\left(\frac{nha}{\xi^2a^2}\right)^2. \tag{5}$$

When the function is

$$\frac{\dot{x}J_0(\dot{x})}{J_1(\dot{x})}\equiv F(\dot{x}), \tag{6}$$

then the following relations can be obtained.

$$\frac{\dot{x}J_1'(\dot{x})}{J_1(\dot{x})}=F(\dot{x})-1 \tag{7}$$

$$\frac{J_2'(\dot{x})}{\dot{x}J_2(\dot{x})}\equiv\frac{1}{2-F(\dot{x})}-\frac{2}{\dot{x}^2}. \tag{8}$$

Using this $F(\dot{x})$ function, the characteristic equations for the first three lossy modes, TM_0, HY_1 and HY_2 series are written as:

$$TM_0:\ ka\frac{Z_\perp}{Z_0}=-jF(\dot{x}) \tag{9}$$

$$HY_1:\ ka\frac{Z_\perp}{Z_0}=j\left(\frac{1}{(\dot{x})^2}\{F(\dot{x})-1\}+\frac{1}{F(\dot{x})-1}\left\{\frac{1}{(ka)^2}-\frac{1}{(\dot{x})^2}\right\}\right)^{-1} \tag{10}$$

$$HY_2:\ ka\frac{Z_\perp}{Z_0}=j\left(\frac{1}{2-F(\dot{x})}-\frac{2}{(\dot{x})}+\frac{2}{\frac{(\dot{x})^2}{2\{2-F(\dot{x})\}}-1}\left\{\frac{1}{(ka)^2}-\frac{1}{(\dot{x})^2}\right\}\right)^{-1}. \tag{11}$$

Numerical Calculations

Although there are large numbers of unwanted modes in the practical TE_{01} transmission line, it has been found that only modes having a smaller value of angular mode index can have a relatively large coupling to the TE_{01} mode.[1] Thus the numerical calculations have been performed only for the TM_0, HY_1, and HY_2 modes using a Fuji 128 relay computer.

As a first step the values of $F(\dot{x})$ are determined in the region $0\leq\rho\leq 10$ for 0°, 5°, 10°, 15°, 20°, and 25° respectively, where

$$\dot{x}\equiv\xi a\equiv\rho e^{-j\phi}. \tag{12}$$

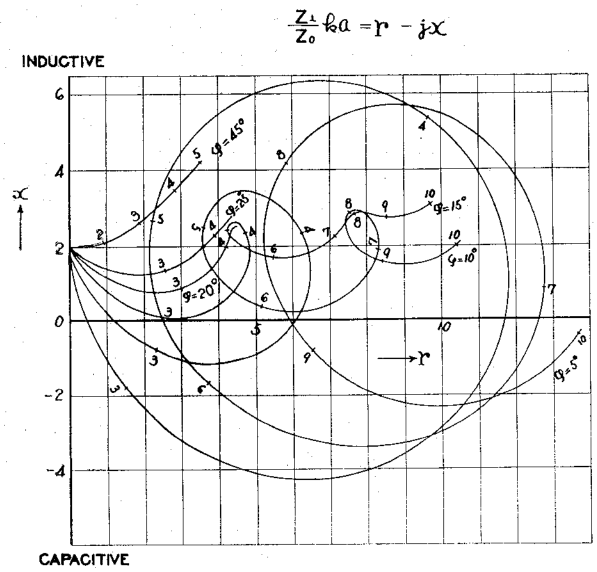

Fig. 2—Surface impedance. The curves show equi-φ lines and the numbers along this line show ρ values.

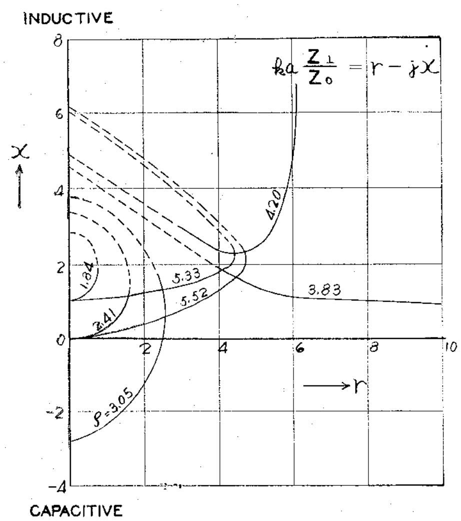

Fig. 3—Surface impedance vs the equiamplitude ρ lines.

These results lead to the normalized surface impedance $kaZ_\perp/Z_0$ for the TM_0 modes. The vector diagrams are shown in the following two figures, that is, the equiphase lines in Fig. 2 (the constant ϕ lines), and the equiamplitude lines in Fig. 3 (the constant ρ lines). As a second step, the propagation constants

$$ha = \beta a + j\alpha a = \sqrt{(ka)^2 - (\xi a)^2} \tag{13}$$

are calculated for the same amplitudes ρ and angles ϕ. The curves of αa vs ρ with the parameter ϕ are given in Fig. 4.

By eliminating the parameter ξa from the above two figures, the relation between the normalized surface impedance $ka\, Z_1/Z_0$ and the attenuation factor αa is found. The results show that there is a large number of TM_0 modes corresponding to any special surface impedance. In order to distinguish them, it is convenient to use the value of the radial propagation constant itself.

In general, unwanted modes with lower attenuation may be more harmful than ones with higher attenuation. Therefore we choose the one mode which shows

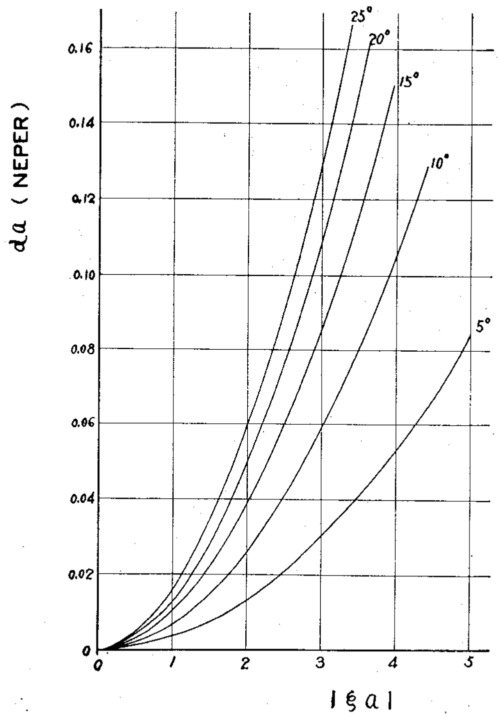

Fig. 4—$|\xi a| \equiv \rho$ vs αa at $ka = 26.2$.

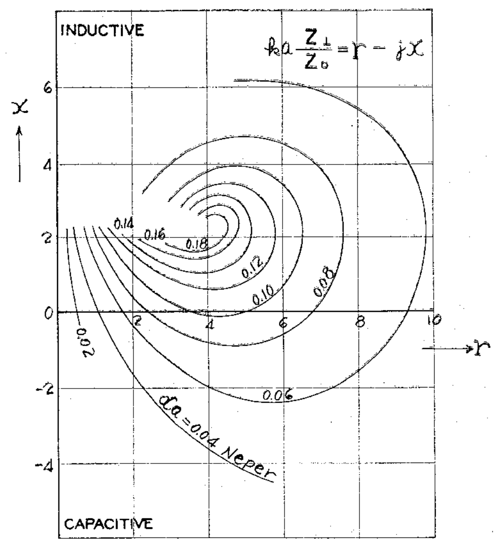

Fig. 5—Relations between surface impedance and attenuation constants of TM_0 waves $ka = 26.2$ (diameter 50 mm, frequency 50 kmc).

minimum attenuation in the infinite series of TM_0 modes corresponding to each given surface impedance. The equiattenuation lines in the $ka\ Z_\perp/Z_0$ plane, as shown in Fig. 5, is plotted for the case of $ka = 26.2$ which corresponds to $2a = 5$ cm and $f = 50$ kmc. In Fig. 5, the attenuation factor αa takes a maximum value 0.2 neper at $ka\ Z_\perp/Z_0 = 4 - j2$. Therefore if a helix waveguide having this optimum value of surface impedance is designed no TM_0 modes in the guide can have an attenuation factor smaller than 0.2 neper, the minimax value of the attenuation factor.

In the same way, for HY_1 and HY_2 modes, we get the relations between $ka\ Z_\perp/Z_0$ and ξ from (8), (9), and (11), which are shown in Figs. 6 and 7 for the case of $ka = 26.2$. The best values of the surface impedance for

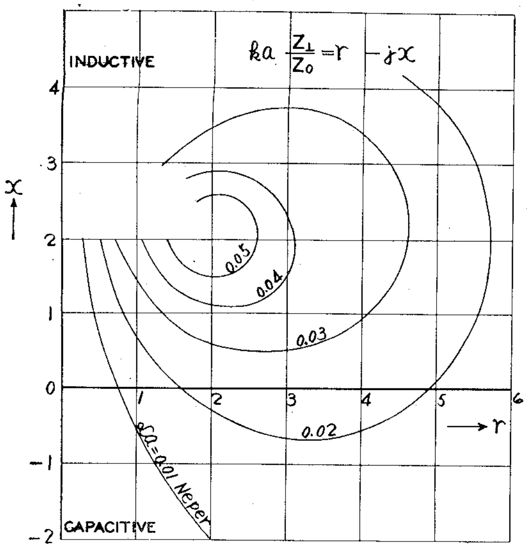

Fig. 6—Relations between surface impedance and attenuation constants of HY_1 waves, $ka=26.2$, (diameter 50 mm, frequency 50 kmc).

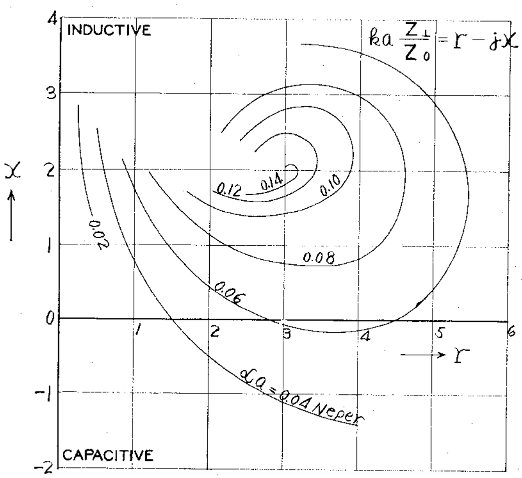

Fig. 7—Relations between surface impedance and attenuation constants of HY_2 waves, $ka=26.2$, (diameter 50 mm, frequency 50 kmc).

TABLE I

THE BEST VALUE OF SURFACE IMPEDANCE AND THE MINIMAX VALUE OF ATTENUATION CONSTANT FOR A HELIX WAVEGUIDE (50 mm ID, at 50 kmc)

Mode	Best Value of Z_1	Minimax Value of α
TM_0	57.6−j28.8 (ohms)	8.0 (nep/meter)
HY_1	28.8−j28.8	2.4
HY_2	43.2−j28.8	6.0

a mode-filter for HY_1 and HY_2 modes may be found from these curves.

Thus we have the best value, as is shown in Table I.

Waveguide Structures

In the previous section, the best value of the surface impedance $Z_\perp$ for the mode filter was found. There remains, however, the problem of designing the physical structures corresponding to this optimum surface impedance. Although this problem has not yet been solved completely, two possible structures are suggested.

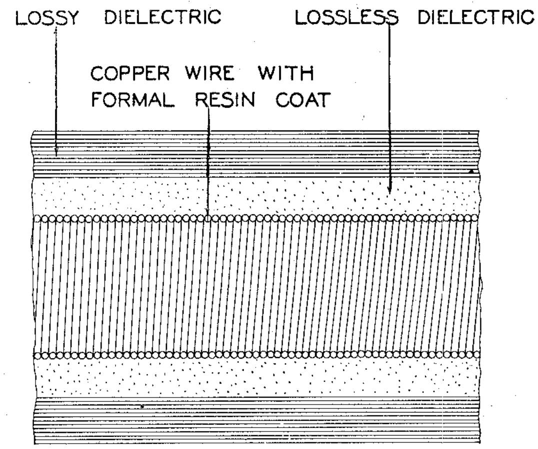

Fig. 8—The longitudinal section of a helix waveguide.

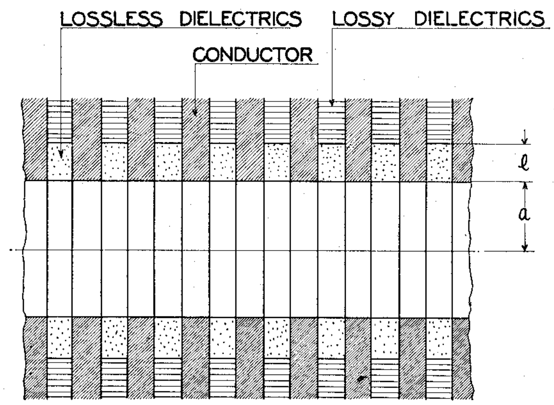

Fig. 9—The longitudinal section of a spaced-ring waveguide.

The first is a sheath helix structure with multilayer jacket, as indicated in Unger's paper.[9] In Fig. 8, the simplest one of this type is shown. The inner layer is lossless dielectrics while the outer jacket is lossy material. Both the thickness of the inner layer and the electrical properties of the outer jacket are varied so as to realize the best value of surface impedance.

Another example is the spaced-ring structure shown in Fig. 9. Here also the depth l of the lossless dielectrics and the lossy material are varied to obtain the best condition. In this case, the radial transmission line calculations can be used to design the surface impedance.

Conclusion

The characteristics of a helix waveguide as a mode-filter has been theoretically examined. The relations between the anisotropic surface impedance and the attenuation constant were investigated to determine the best value of the above impedance for a mode filter. The best values were obtained numerically for the TM_0, HY_1, and HY_2 modes. Two examples of helix structure which have possibility of giving the best value of the surface impedance were also proposed.

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